

## MAHARASHTRA STATE BOARD OF VOCATIONAL EDUCATION EXAMINATION, MUMBAI

## Examination—April, 2018 (Two Year Diploma Courses)

## Group—All Groups

[TIME ALLOWED — 3 HOURS]

(MARKS — 70)

## APPLIED MATHEMATICS (THEORY)

*Notes.*— (1) All questions are *compulsory*.(2) Figures at right indicate *full* marks.**Marks**1. (a) Fill in the blanks (any *twelve*) :—

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(i) If  $y = a^x$  then  $\frac{dy}{dx} = \dots\dots\dots$ .

(ii)  $\int \tan^{-1} x \times dx = \dots\dots\dots$ .

(iii) Find the value of  $\log_{0.5} \left( \frac{\sqrt{32}}{1024} \right) = \dots\dots\dots$ .

(iv) If  $f(x) = f(2x+1)$  for  $f(x) = x^2 - 3x + 4$ , then find value of  $x$ ;  $x = \dots\dots\dots$ .

(v) Modulus and amplitude of number 40- 91 are..... and ..... respectively.

(vi)  $15^2 + 16^2 + 17^2 + 18^2 + \dots\dots\dots + 49^2 + 50^2 = \dots\dots\dots$ .

(vii)  $\binom{7}{0} + \binom{7}{1} + \binom{7}{2} + \binom{7}{3} + \binom{7}{4} + \binom{7}{5} + \binom{7}{6} + \binom{7}{7} - 2 = \dots\dots\dots$ .

(viii)  $\lim_{x \rightarrow 4} \frac{(x^4 - 1024)}{(x - 16)} = \dots\dots\dots$ .

(ix)  $\frac{d}{dx} (\sec x \cdot \tan x) = \dots\dots\dots$ .

(x)  $\int \sin^3 x \, dx = \dots\dots\dots$ .

(xi) Complete the DMorgan's law  $(A \vee B)' = \dots\dots\dots$ .

(xii) ..... is the range of the  $2\cos(x/4)$  function.

(xiii) ..... is centre and ..... is radius of the circle  $x^2 + y^2 - 2x + 4y - 3 = 0$ .

(xiv) The vectors  $3i - 5j + k$  and  $4i - 2j + pk$  are collinear then value of  $p = \dots\dots\dots$ .

(xv) ..... is the equation of tangent to parabola  $x^2 = 20y$  at point (6,2)

(b) Solve the following (Any four) :—

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(i) Find the cofactor of the elements of the following matrix.

$$\begin{bmatrix} 2 & 4 \\ -8 & -3 \end{bmatrix}$$

(ii) Find the joint equation of the lines  $7X - 3Y = 0$  and  $4X + 9Y = 11$ .

(iii) Find the direction ratios and direction cosines of the line passing through A(-6, 5, 3) and B(-1, -3, 7).

(iv) Find  $\frac{dy}{dx}$  if  $y = \sin(x^2 + x + 1)$

(v) Integrate the following function with respect to x

$$\frac{\cos x}{(1 + \cos x)}$$

2. Attempt any two of the following :—

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(a) Discuss the continuity of the following function in its domain

$$f(x) = 2x \quad \text{for } x \geq 0$$

$$f(x) = 2x^2 \quad \text{for } x < 0$$

(b) Evaluate

$$(i) \int \frac{108}{\sqrt{5x+3} - \sqrt{5x-3}} dx$$

$$(ii) \int_{-2}^1 \frac{1}{x^2 + 4x + 13} dx$$

(c) If a line makes angles  $30^\circ$ ,  $45^\circ$  and  $135^\circ$  with the X, Y and Z axes respectively, then find its direction cosines.

(d) Find the equation of tangent to the circle  $x^2 + y^2 = 100$  at (6,-8). Also find the equation of tangent parallel to this tangent.

3. Attempt any two of the following :—

12

(a) Using truth table examine the following statement patterns are tautology, contradiction or contingency :—

$$(i) (p \vee q) \wedge (p \vee r)$$

$$(ii) (\sim p \wedge q) \vee (q \wedge r)$$

(b) Prove that :—

$$(i) \frac{\sec 4A - 1}{\sec 2A - 1} = \frac{\tan 4A}{\tan 2A}$$

$$(ii) \sin^4 x + \cos^4 x = 2 \sin^2 x - 1$$

(c) Solve the differential equation  $\frac{dy}{dx} + y = e^{-x}$

(d) Differentiate with respect to x :  $\sin^{-1}(x^2 - 1)$

4. Give brief answers (Any *two*) :— 14
- (a) Find the equation of normal to circle  $x^2 + y^2 = 100$  at  $(-6, 8)$
- (b) Find the area of the region lying between  $y = 2x^2$  and lines  $x=1$  and  $x=4$ .
- (c) Solve following simultaneous equations by any matrix method :—  
 $2X - Y + Z = 1$  ;  
 $X + 2Y + 3Z = 8$  ;  
 $3X + Y - 4Z = 1$
- (d) Simplify :—  
 (i)  $\frac{11 - 3i}{2i + 5}$   
 (ii)  $\frac{12 + 5i}{13 - 12i} + \frac{12 - 5i}{13 + 12i}$
5. Attempt any *two* of the following (Any *four*) :— 16
- (a) Bismuth has half life of four days. A sample originally has 2000mg. Find the mass of the remaining after 30 days.
- (b) Find the Harmonic Mean (H. M.) of two positive numbers whose A. M. is 29 and G. M. is 10.
- (c) Find  $r$  if  $\binom{14}{2r} : \binom{10}{2r-4} = 143:10$
- (d) Find the equation of locus of points such that the sum of distance from  $(4, 0)$  and  $(-4, 0)$ .
- (e) A pair of dice is thrown. If sum of the numbers is an odd number, what is the probability that it is a perfect square ?
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